Power-domination in triangulations

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Universidad de Cadiz

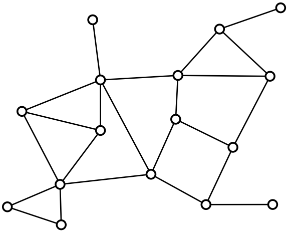
JGA, Orléans, 6 novembre 2015

Power domination

Control ~~the world~~ a system with a minimal number of captors [Baldwin et al. '91, '93]

- Some vertices in a starting set S (captors).
- $N[S] = M$
- (propagation step) $u \in M$. If $v \in N(u)$ is the only vertex outside $N[u] \cup M$: $M \to M \cup \{x\}$.

S is a power dominating set (PDS) if $M = V(G)$ at the end.
$\gamma_P(G)$ (power domination number of G): minimum size of a PDS.



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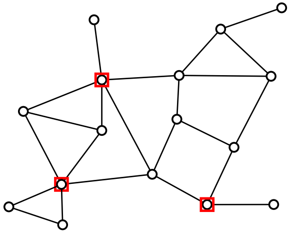

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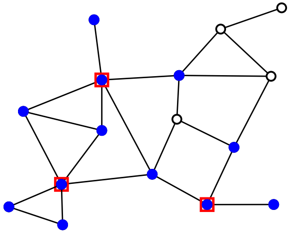

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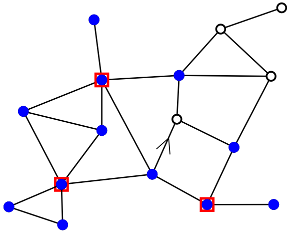

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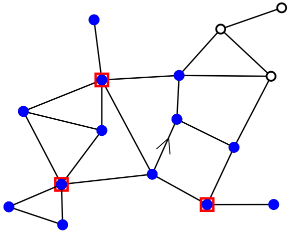

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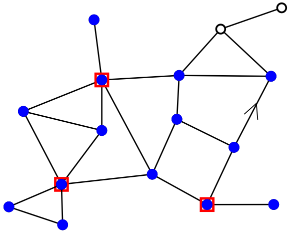

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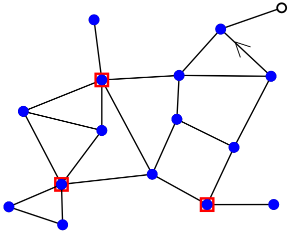

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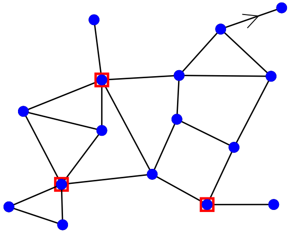

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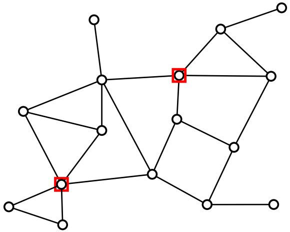

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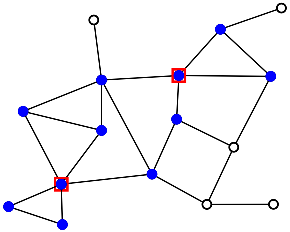

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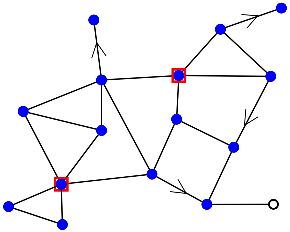

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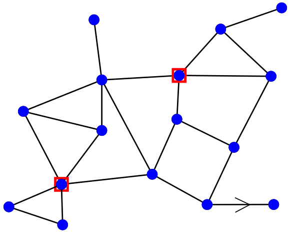

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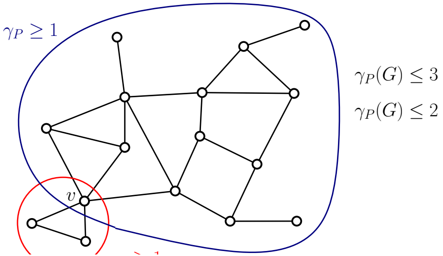

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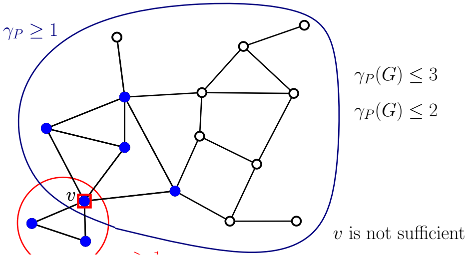

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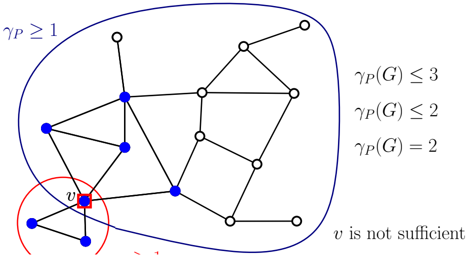

Some known results

Power-dominating set

Input: A (undirected) graph $G = (V, E)$, an integer $k \geq 0$.
Question: Is there a power-dominating set $S \subseteq V$ with $|S| \leq k$?

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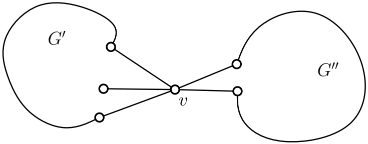

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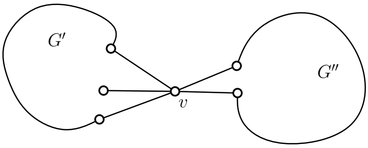

→ restrict to triangulations: no cut-vertex!

Power-domination in triangulations

[Matheson & Tarjan '96]

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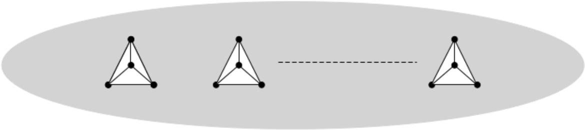

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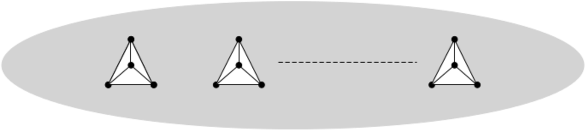

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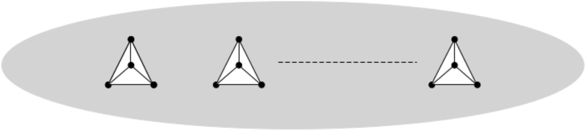

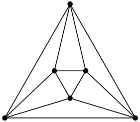

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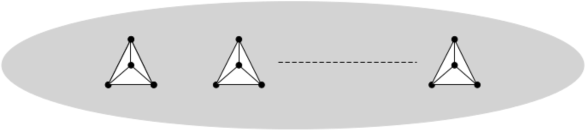

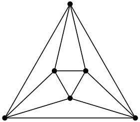

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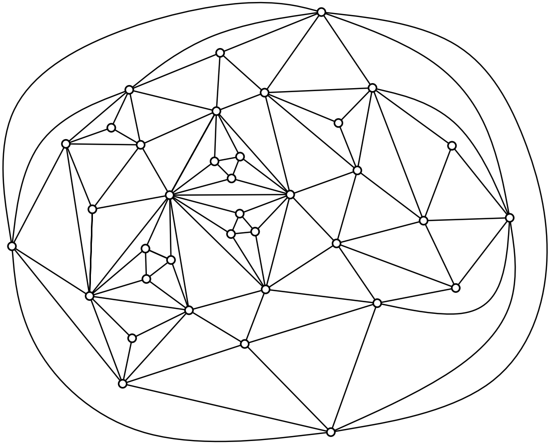
Main Theorem

$$\gamma_P(G) \leq \frac{n-2}{4} \text{ if } G \text{ is a triangulation with } n \geq 6 \text{ vertices.}$$

Our algorithm

- Monitor the octahedrons, and propagate.
- For every vertex v in $\overline{M}$ in decreasing degree (in G) order: if adding v to S adds at least 4 vertices in M (with propagation): add v to S.
  No vertex selected: stop.



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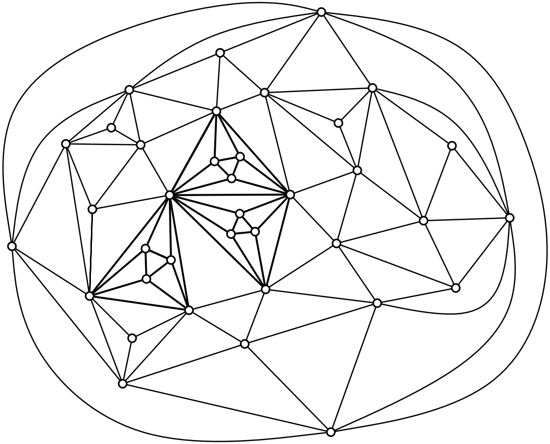

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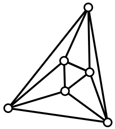

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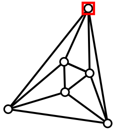

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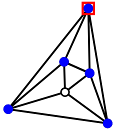

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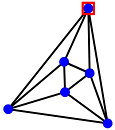

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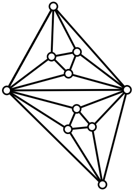

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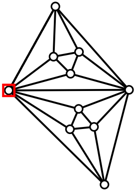

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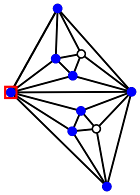

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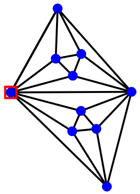

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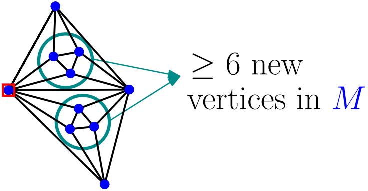

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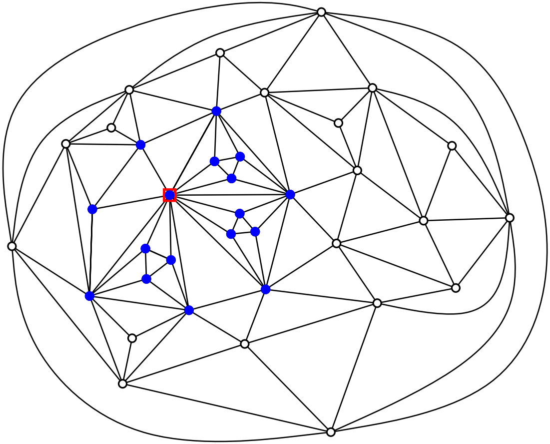

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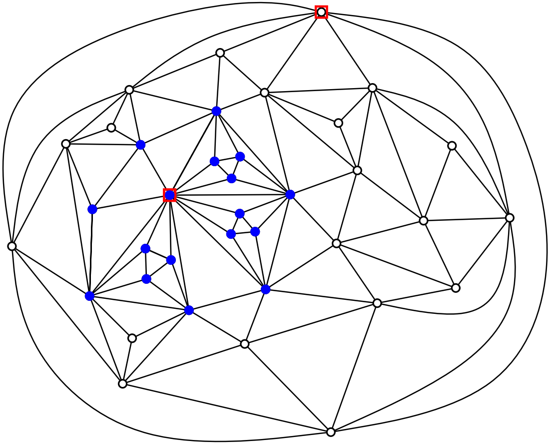

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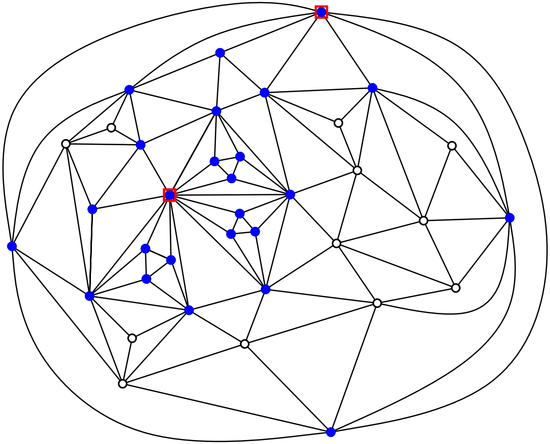

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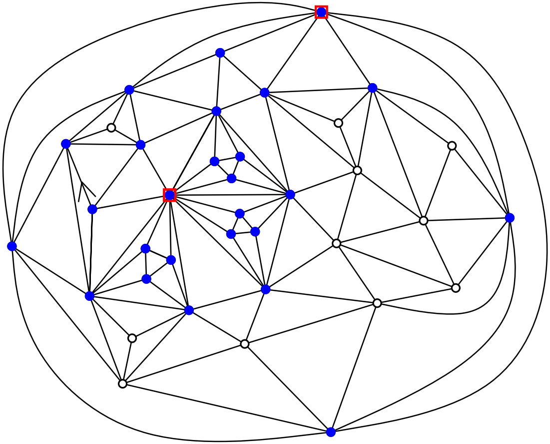

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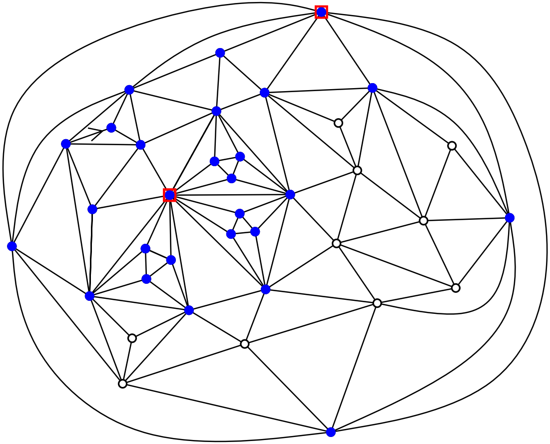

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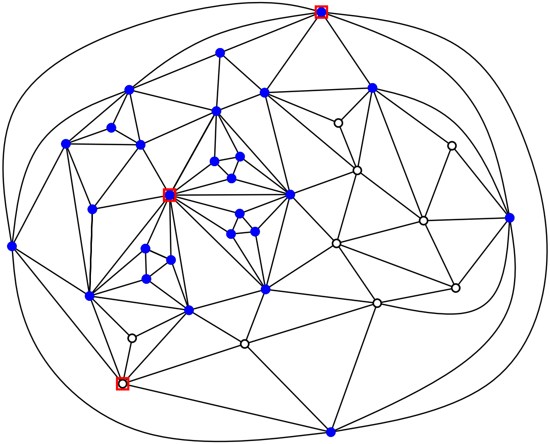

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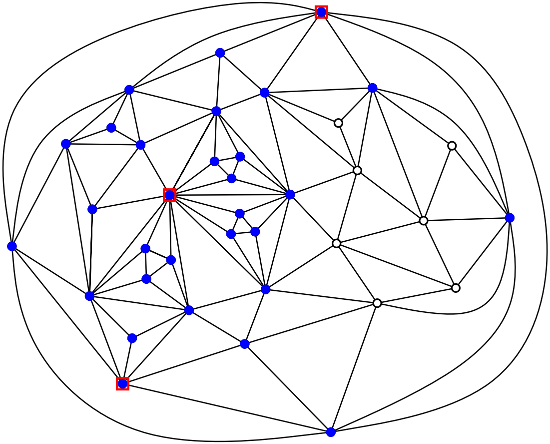

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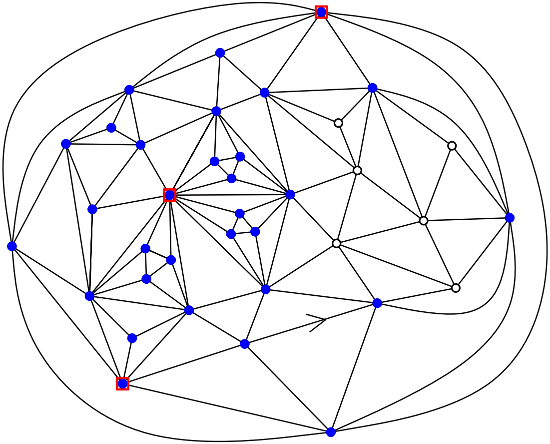

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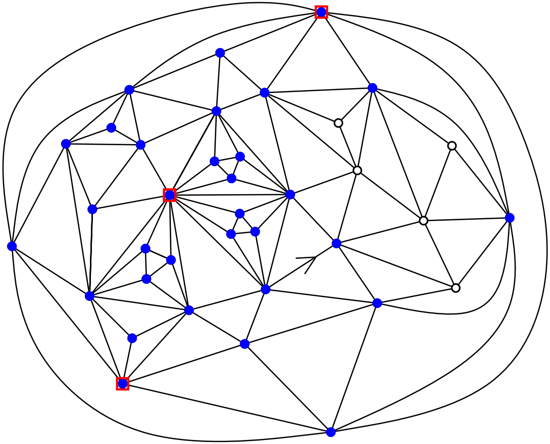

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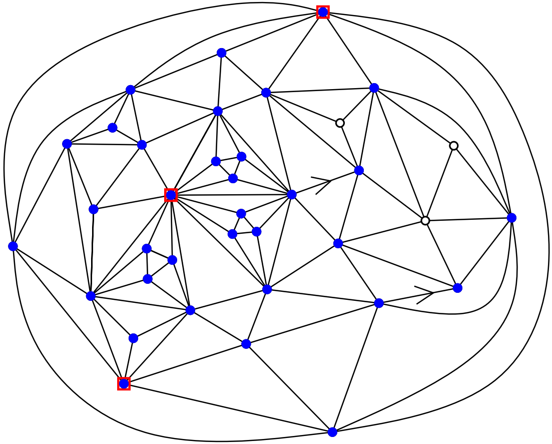

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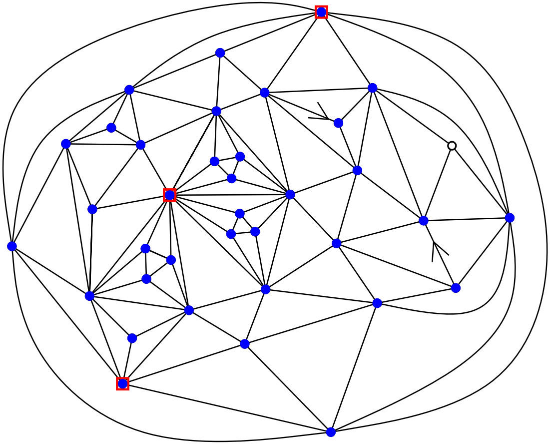

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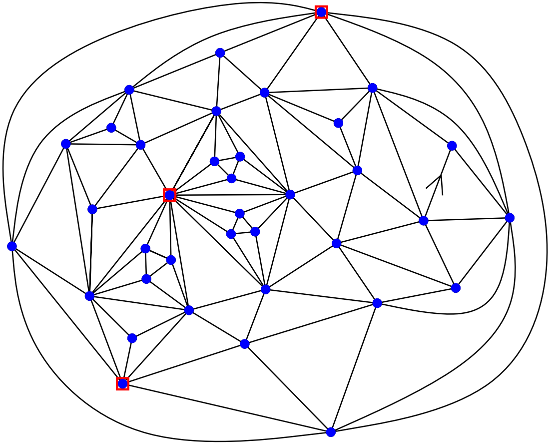

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The graph $G[\overline{M}]$ satisfies the following properties:

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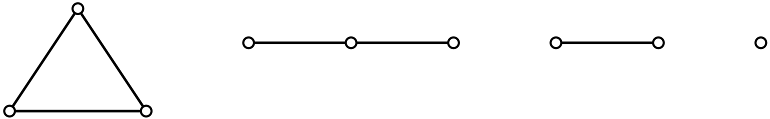

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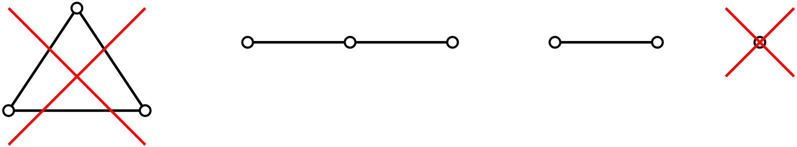

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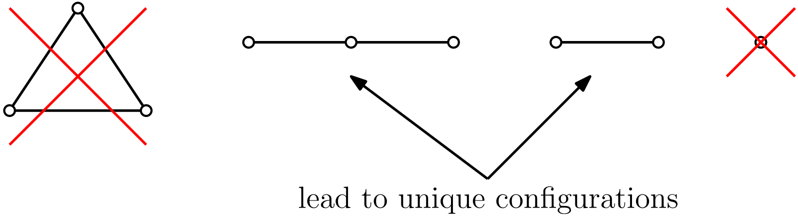


Connected components of $G[\overline{M}]$

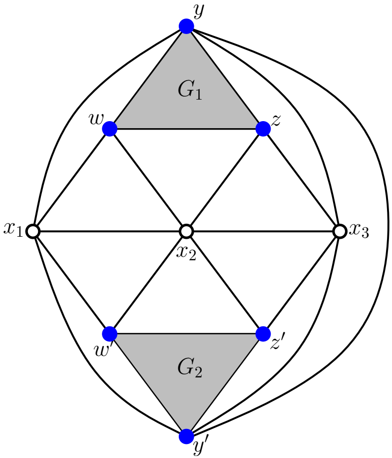
Global technique used for all cases: try to build G around the hypothetical connected component.

(Some) Tools used in this (long) proof:

- planarity (contradiction with Euler's formula)
- contradiction with the conditions to choose a vertex in S : maximal degree or contribution of each vertex
- induction reasoning

Induction example: P_3

If a connected component of $G[\overline{M}]$ is isomorphic to P_3, then G is isomorphic to:



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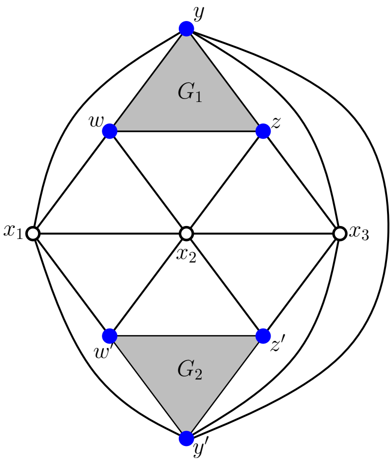

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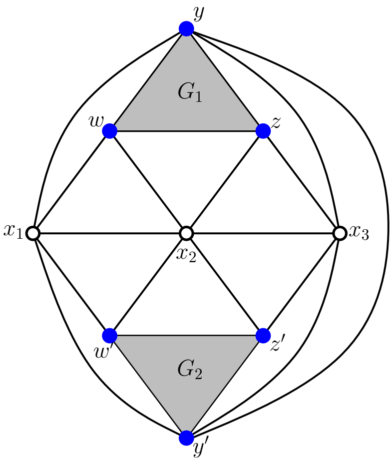

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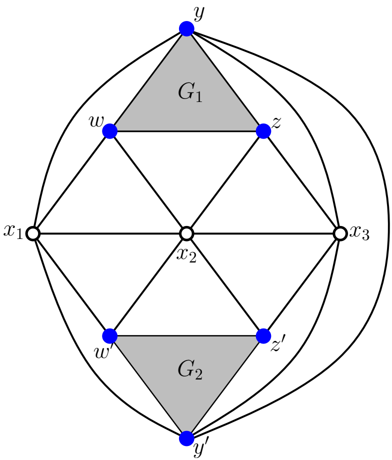

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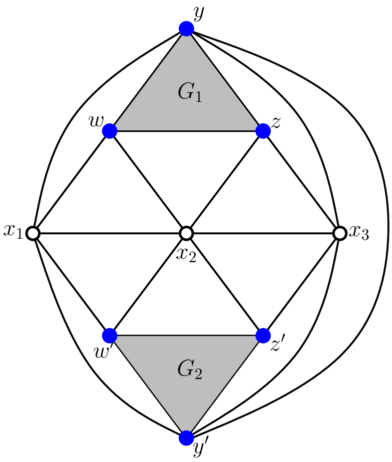

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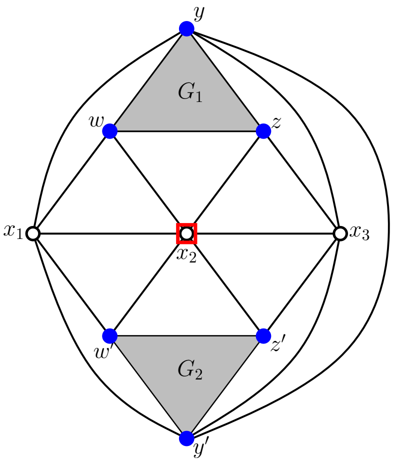

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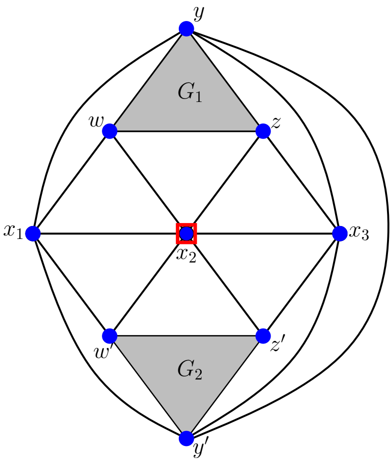

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