Power domination in triangulations

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Joint work with Paul Dorbec and Antonio Gonzalez

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Universidad de Cadiz

GT Graphes et optimisation, LaBRI, 8 janvier 2016

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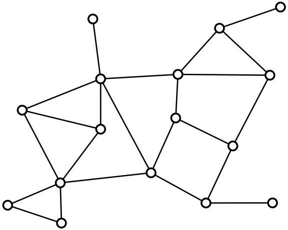
A variant of domination: we can "deduce" things

- Some vertices in a set of "captors" S and $N[S] = M$ (domination)



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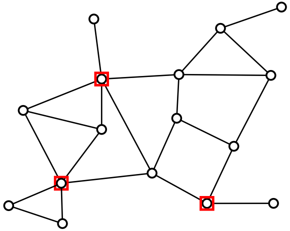

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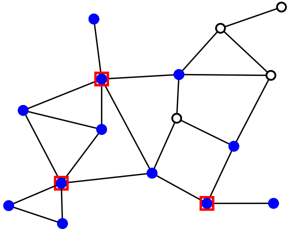

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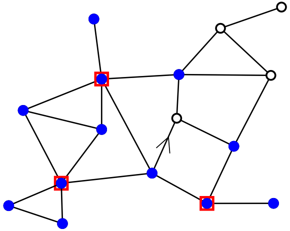

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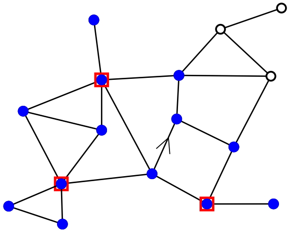

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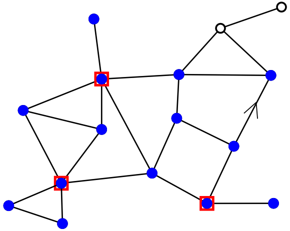

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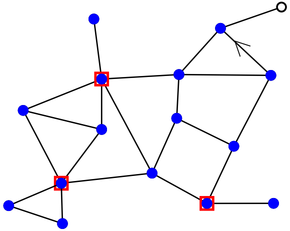

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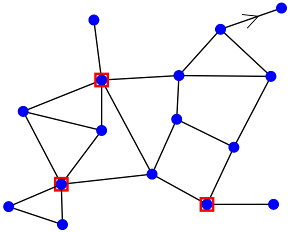

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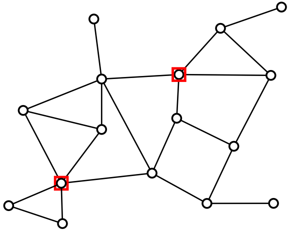

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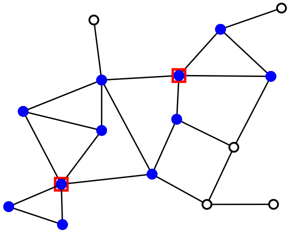

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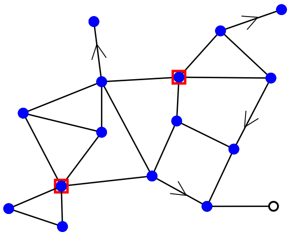

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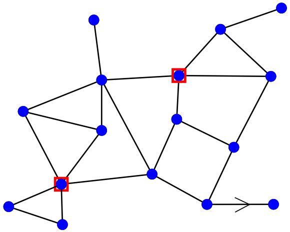

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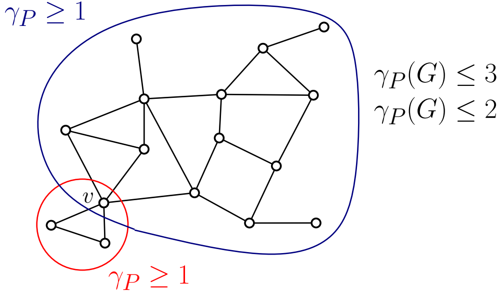

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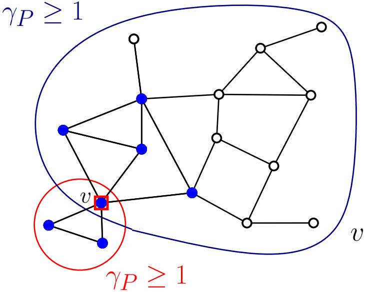

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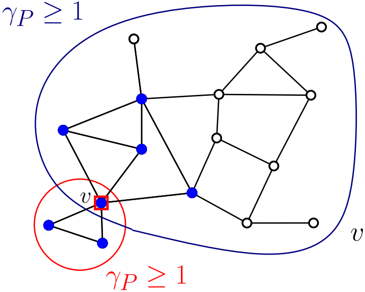

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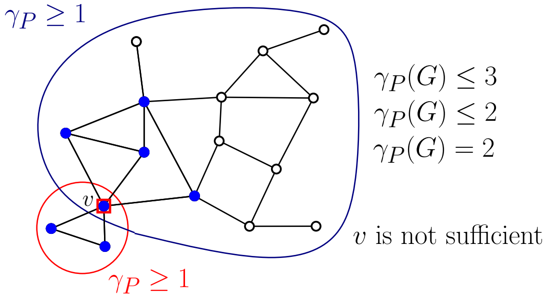
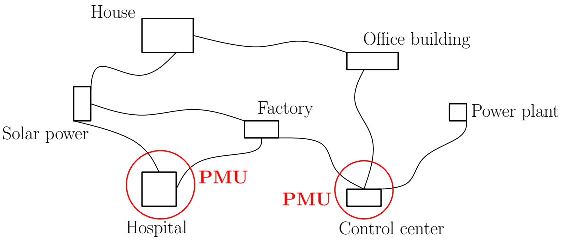


Initially: control an electrical system with a minimal number of captors [Baldwin et al. '91, '93]

Electrical system monitoring

Placing a minimal number of measurement units (PMUs) to monitor an electrical system (devices and cables):



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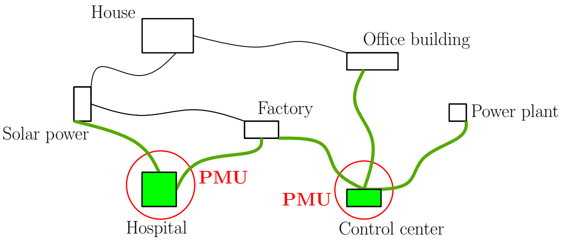

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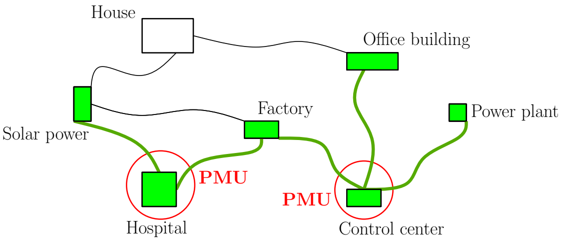

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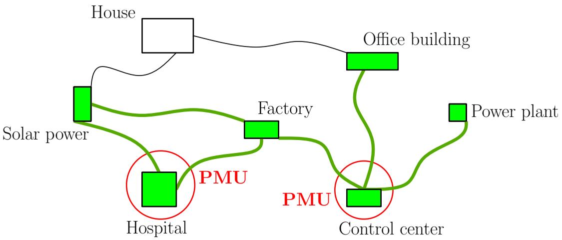

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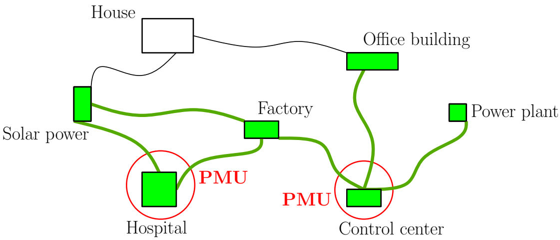

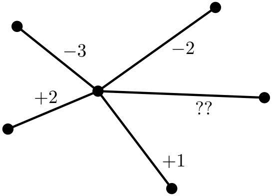

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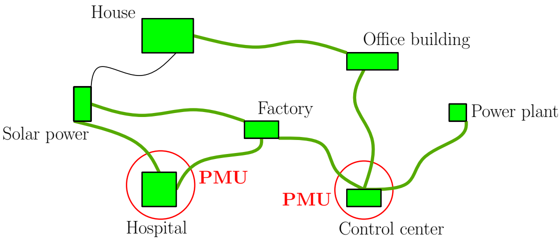

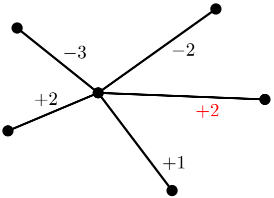

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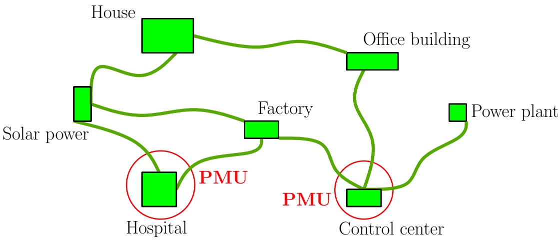

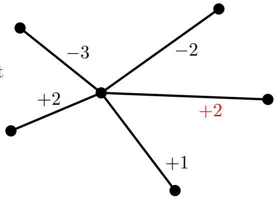

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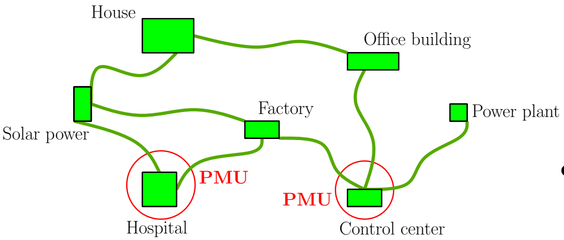

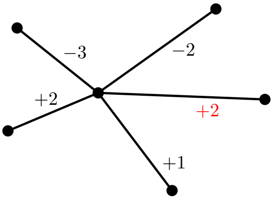

[Haynes et al. '02] Equivalent to monitoring only vertices of a graph (power domination)

Some known results

Power-dominating set

Input: A (undirected) graph $G = (V, E)$, an integer $k \geq 0$.
Question: Is there a power-dominating set $S \subseteq V$ with $|S| \leq k$?

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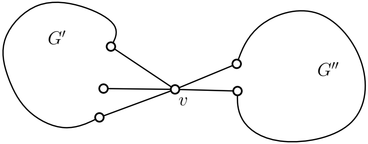

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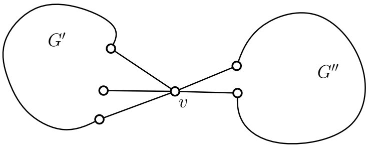

→ restrict to triangulations: no cut-vertex!

(Power)domination in triangulations

[Matheson & Tarjan '96]

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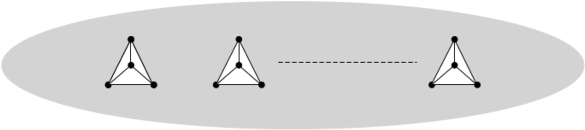

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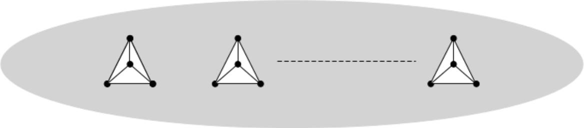

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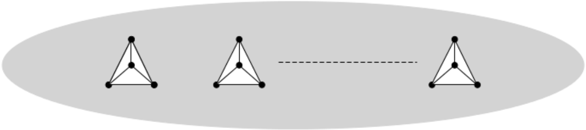

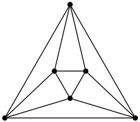

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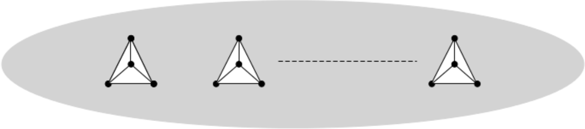

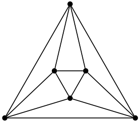

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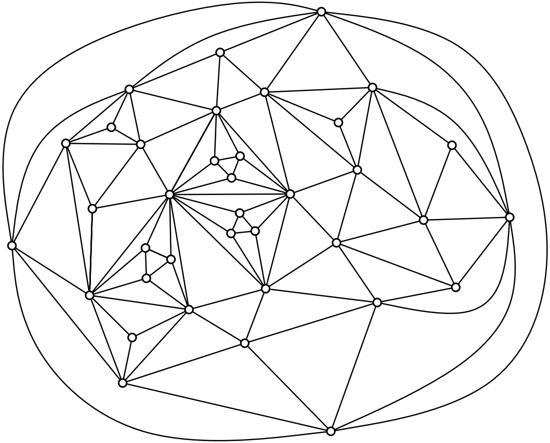
Main Theorem

$$\gamma_P(G) \leq \frac{n-2}{4} \text{ if } G \text{ is a triangulation with } n \geq 6 \text{ vertices.}$$

Our algorithm

- Monitor the octahedrons with $l \leq n'/6$ captors, and propagate.



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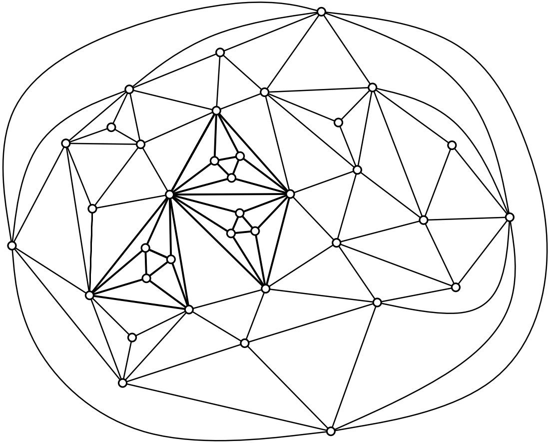

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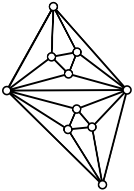

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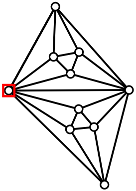

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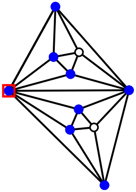

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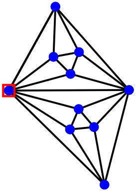

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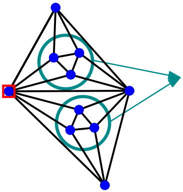

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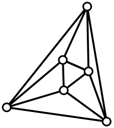

Isolated octahedron:
select a vertex of the outer face in S

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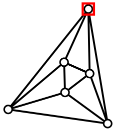

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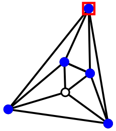

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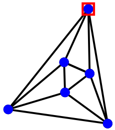

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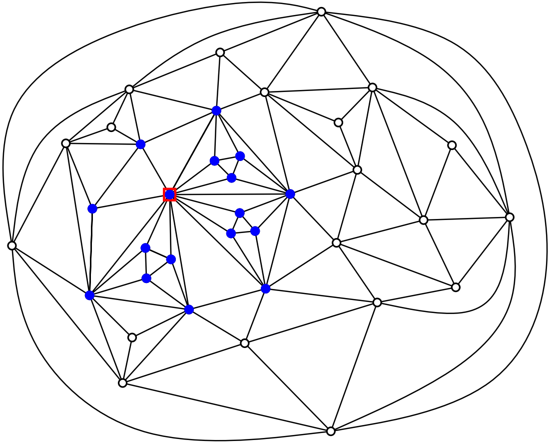

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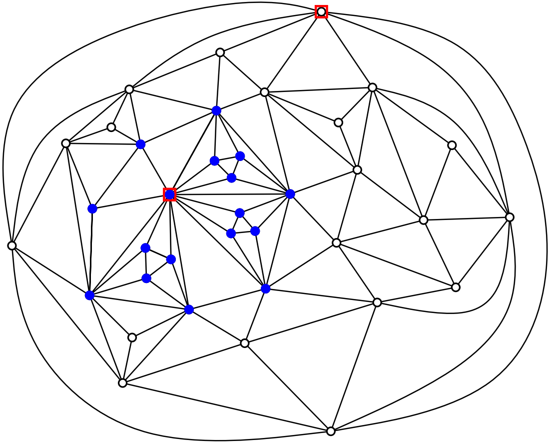

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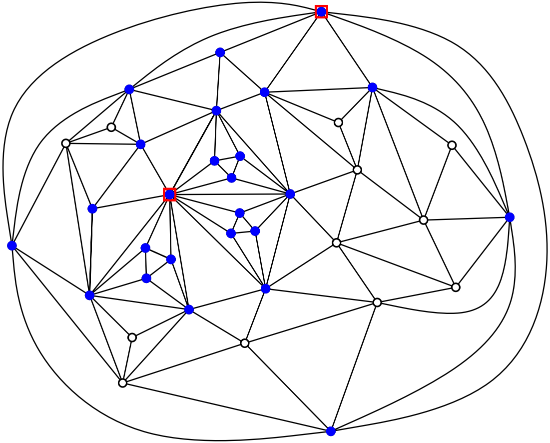

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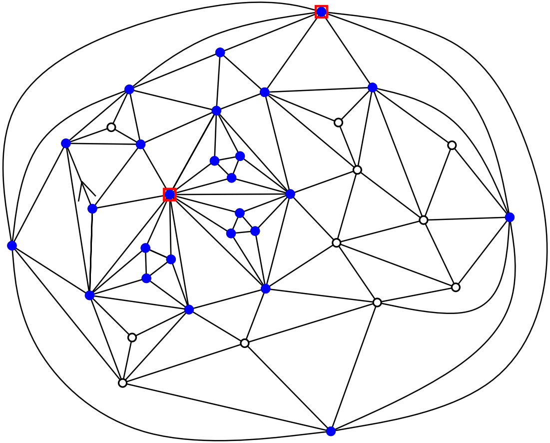

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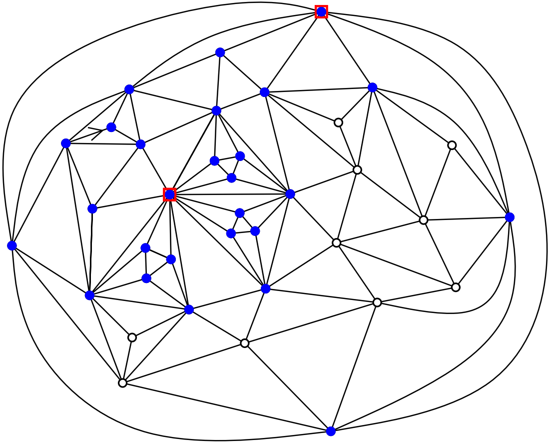

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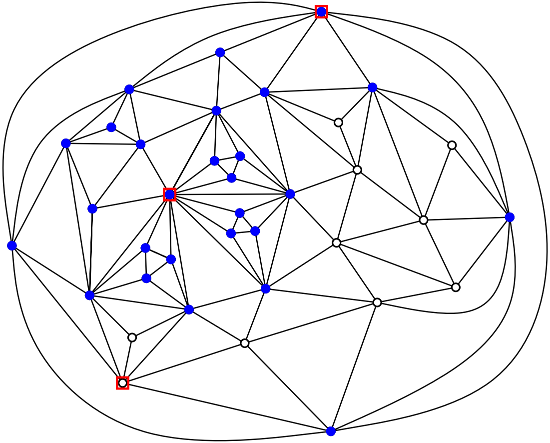

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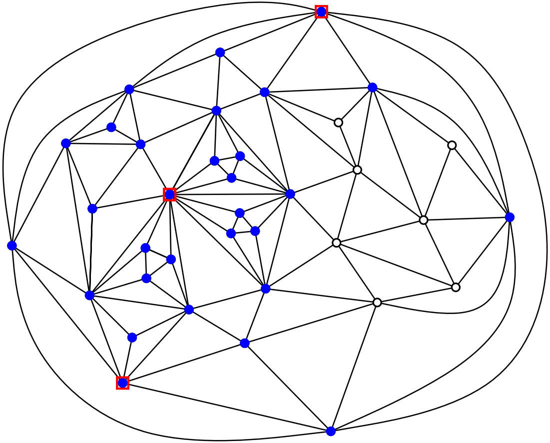

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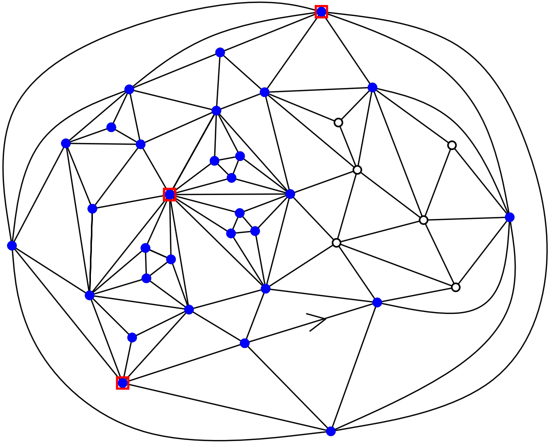

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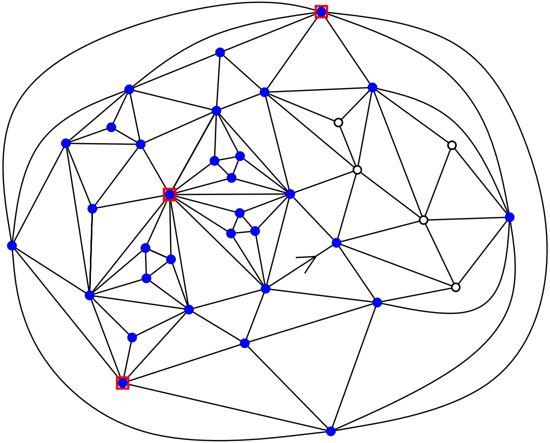

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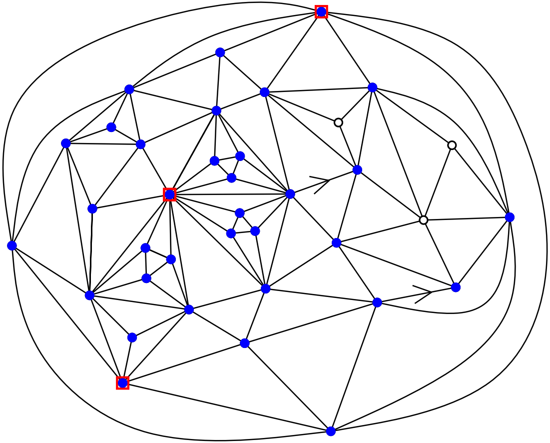

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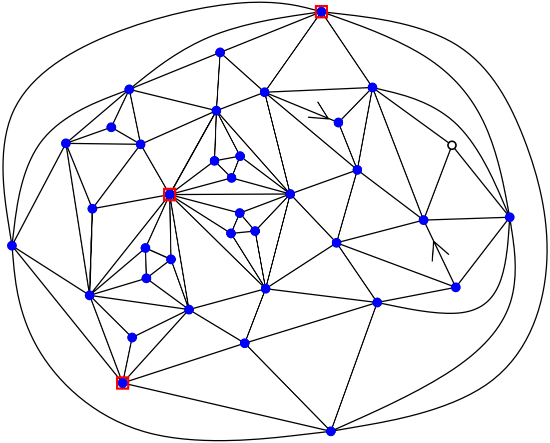

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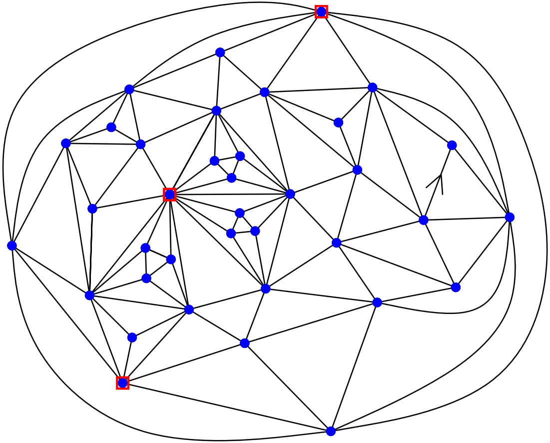

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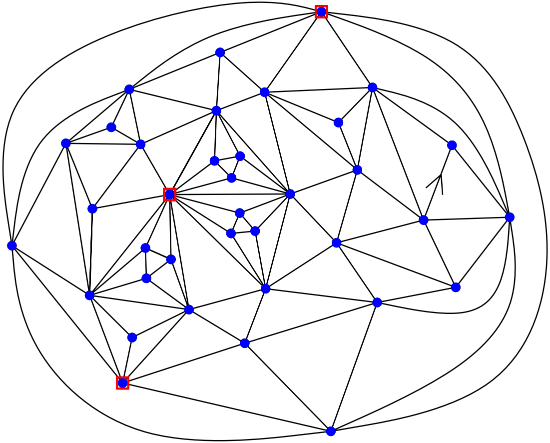

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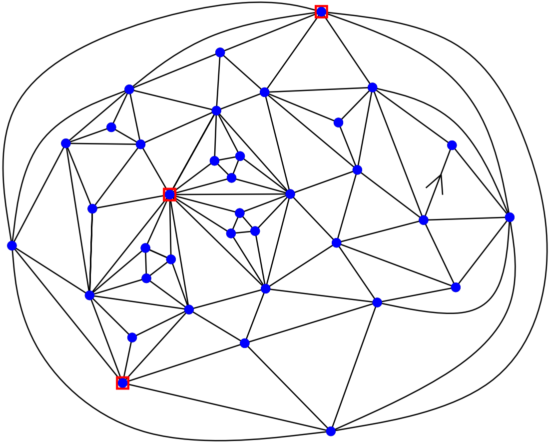

Intuitively: monitor ≥ 6 vertices with the first captor, then ≥ 4 vertices with each captor.
Suppose the graph is not entirely monitored at the end: $G[\overline{M}] \neq \emptyset$

After the main algorithm...

If $G[\overline{M}] \neq \emptyset$, the following properties hold:

(a) $G[\overline{M}]$ has maximum degree at most 2.

(b) Each connected component of $G[\overline{M}]$ has at most three vertices.

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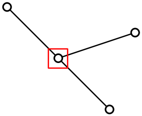

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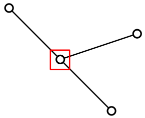

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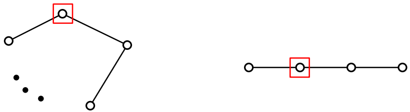

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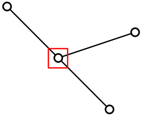

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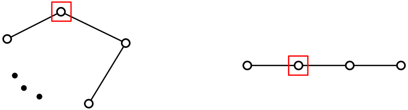
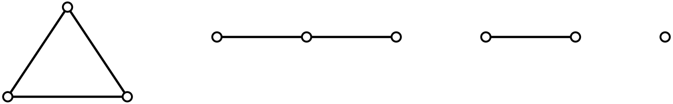


→ A connected component of $G[\overline{M}]$ is isomorphic to K_3, P_3, P_2 or K_1.



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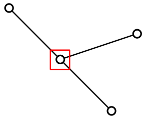

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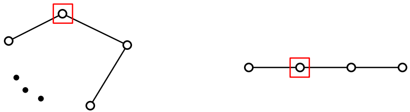

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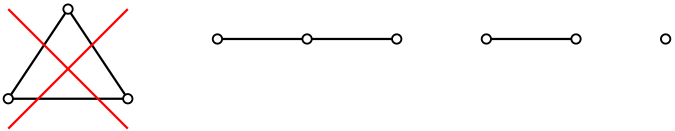

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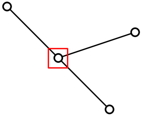

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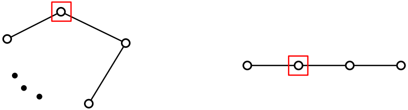

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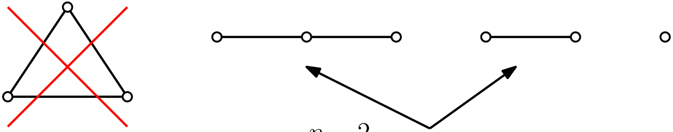

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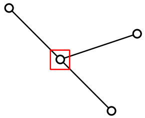

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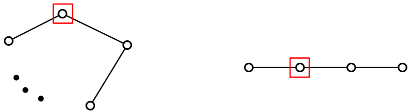

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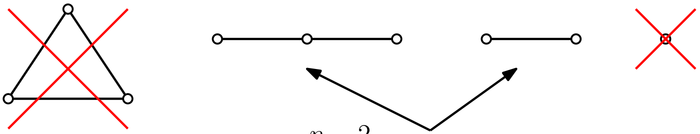

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Connected components of $G[\overline{M}]$

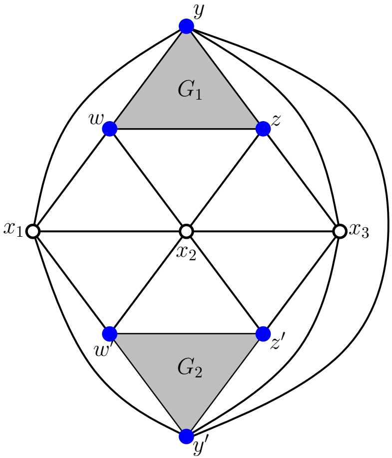
Global technique used for all cases: try to build G around the hypothetical connected component.

(Some) Tools used in this (long) proof:

- planarity (contradiction with Euler's formula)
- contradiction with the conditions to choose a vertex in S : maximal degree or contribution of each vertex
- induction reasoning

Induction example: P_3

If a connected component of $G[\overline{M}]$ is isomorphic to P_3, then G is isomorphic to:



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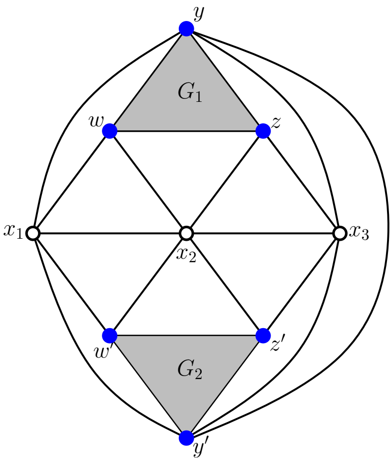

G_1 and G_2 have ≥ 6 vertices (oth. contradiction with the degree condition)

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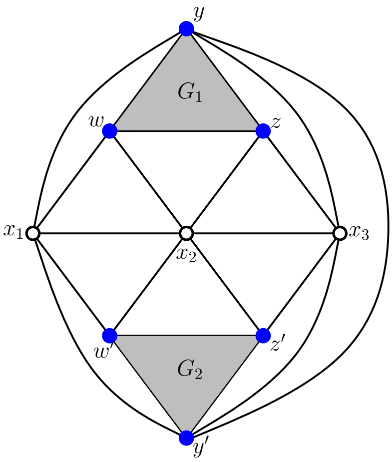

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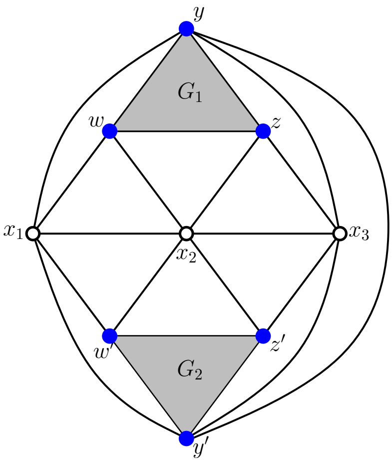

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$\gamma_P(G_1) \leq \frac{n_1 - 2}{4}$

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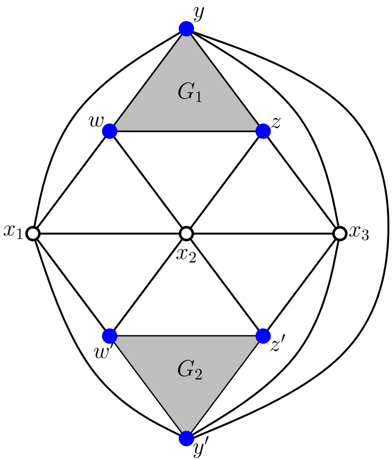

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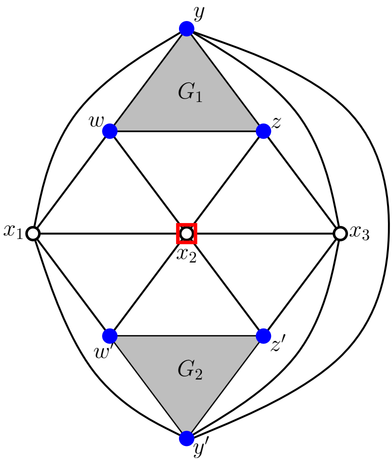

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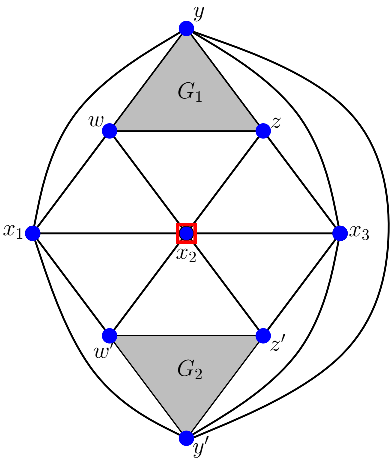

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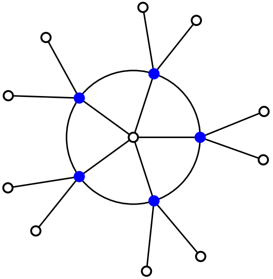
The last case: each vertex x in $\overline{M}$ has the following neighborhood:



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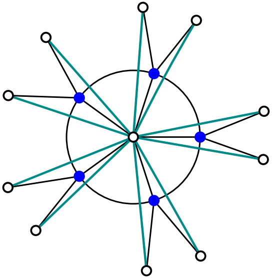

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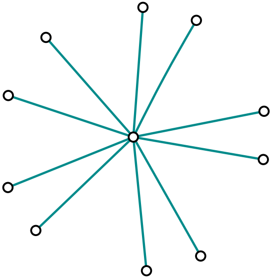

Create an edge between two vertices $u, v \in \overline{M}$ at distance 2
The new graph is planar and every vertex has degree at least 6
(each vertex has deg ≥ 3 in G): contradiction!

And now?

Future work:

- Prove that our algorithm has a linear complexity
- Find (a family of) graphs for which our algorithm reaches the $\frac{n-2}{4}$ bound

Open questions:

- Can we "change the constant factor" in $\frac{n-2}{4}$?
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