Non-aligned drawings of planar graphs

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GT Graphes et Optimisation, LaBRI
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Non-aligned drawings

Drawing **large** graphs with hierarchical view

a vertex in the drawing = a group of vertices in the graph

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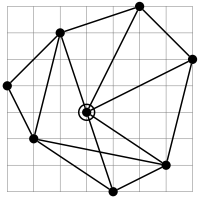

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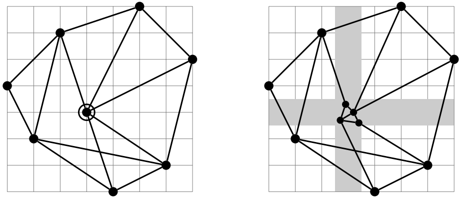

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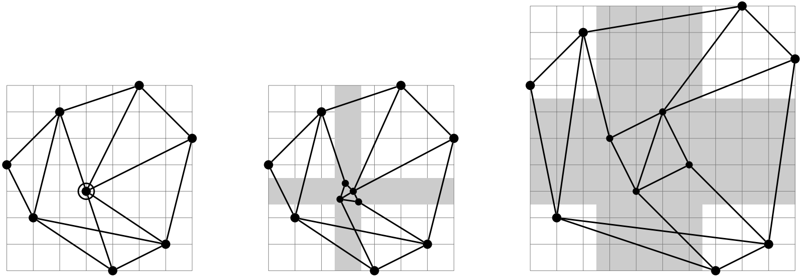

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A non-aligned drawing of a graph with n vertices is:

- on an $f(n) \times g(n)$ grid, for some functions f and g
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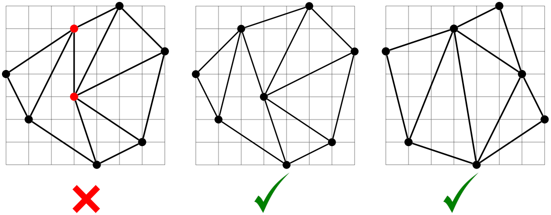

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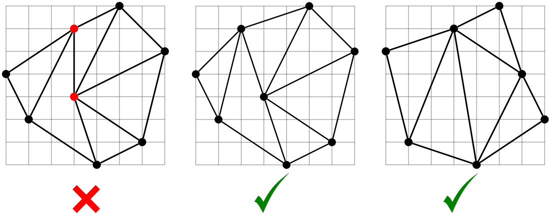

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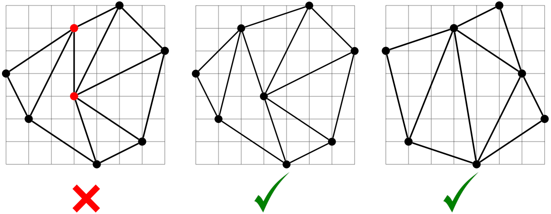

Here: maximal planar graphs (faces are triangles) → **planar** drawings

Edges: "straight-line" or "bend" (on the grid points)

Our results

Every planar graph with n vertices has a:

- non-aligned drawing in a $n \times n$-grid with $\leq \frac{2n-5}{3}$ bends.
  (only 1 if the graph is 4-connected)
- non-aligned straight-line drawing in an $n \times O(n^3)$ grid
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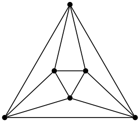

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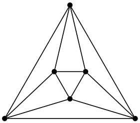

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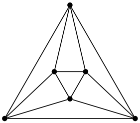

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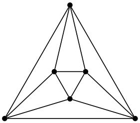

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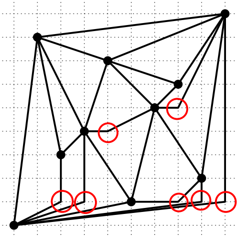

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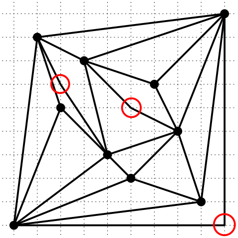

[Biedl, Pennarun '16]

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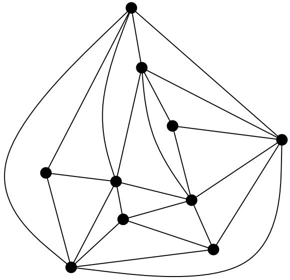
Filled triangle: triangle with some vertices inside
(= **separating triangles** + outerface)



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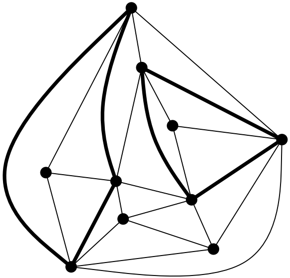

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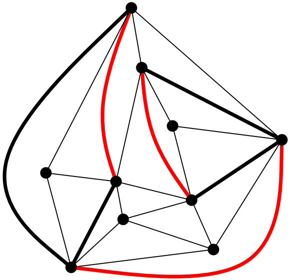
A set of edges E is a independent filled-hitting set of G if every filled triangle of G has an edge within E and the edges of E are not incident.



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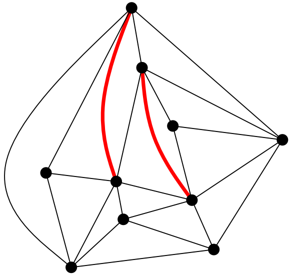

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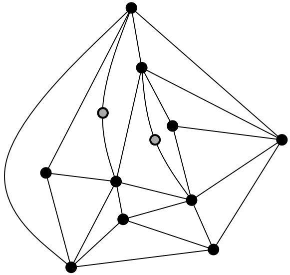

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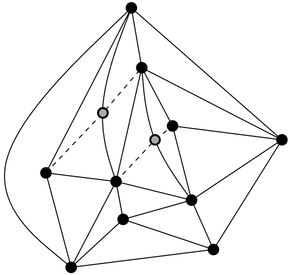

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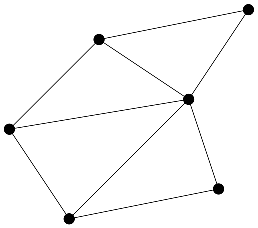
A rectangle-of-influence (RI) drawing:

- a straight-line planar drawing
- the minimum open rectangle containing u and v is empty if (u, v) is an edge



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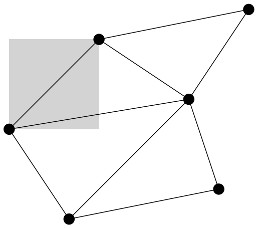

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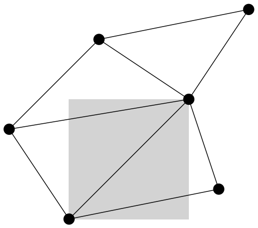

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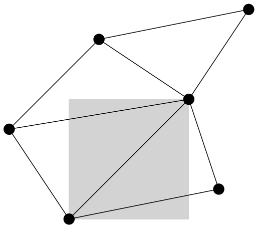
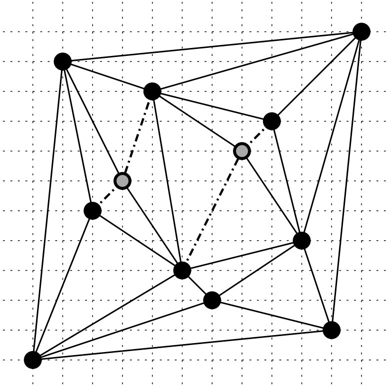


[Biedl et al. '99] If G is 4-connected, and e is an edge of the outerface, then $G - e$ has a planar non-aligned RI-drawing on an $n \times n$ grid.

Non-aligned drawings on an $n \times n$ grid

Non-aligned drawing of $G - e$



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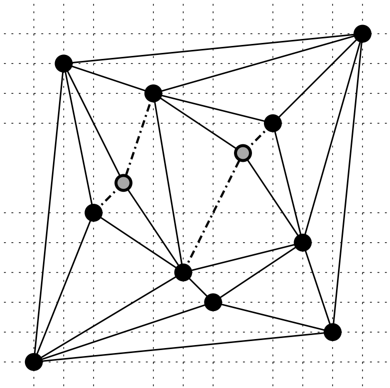
Keep relative orders, original vertices on a $n \times n$ grid and grey ones inbetween



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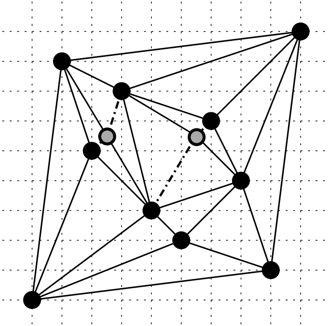

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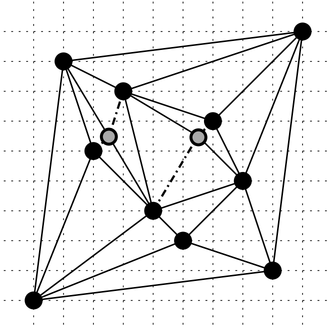

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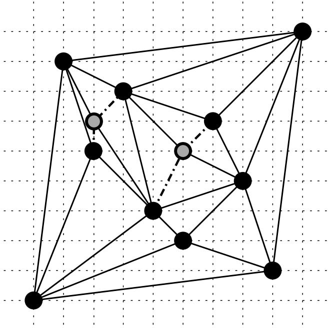

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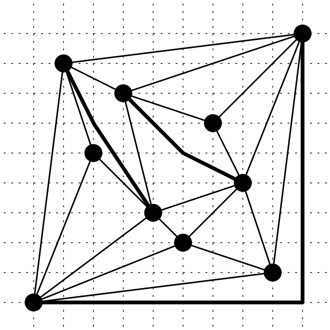

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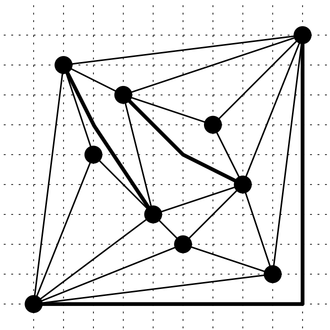

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Every planar graph with n vertices has a non-aligned drawing in an $n \times n$ grid with $\#\{\text{independent filled-hitting set}\}$ bends.

NON-ALIGNED DRAWINGS ON AN $n \times n$ GRID

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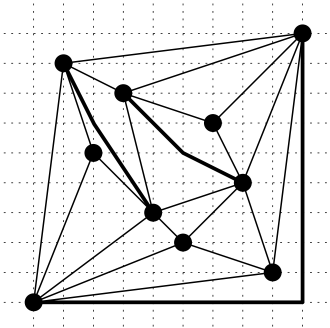

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Our results

Every planar graph with n vertices has a:

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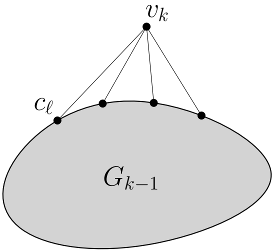
- **non-aligned straight-line drawing in an $n \times O(n^3)$ grid**
- non-aligned straight-line drawing in an $O(n^2) \times O(n^2)$ grid

Non-aligned drawing on an $n \times O(n^3)$-grid

A canonical ordering of a maximal planar graph is a vertex order $v_1 \cdots v_n$ such that the outerface is $[v_1, v_2, v_n]$ and for any $3 \leq k \leq n$, $G_k = G[v_1 \cdots v_k]$ is 2-connected [de Fraysseix, Pach, Pollack '90].

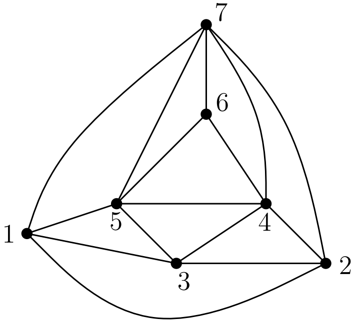
each v_k: predecessors forming an interval on the outerface of G_{k-1}

c_ℓ: left-most predecessor



Non-aligned drawing on an $n \times O(n^3)$-grid

1. Canonical ordering $v_1 \cdots v_n$ of $V(G)$



Non-aligned drawing on an $n \times O(n^3)$-grid

1. Canonical ordering $v_1 \cdots v_n$ of $V(G)$
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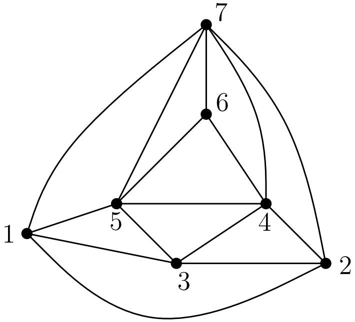
   - $w \neq c_r$ pred. of v_k: $w \to v_k$
   - $v_k \to c_r$



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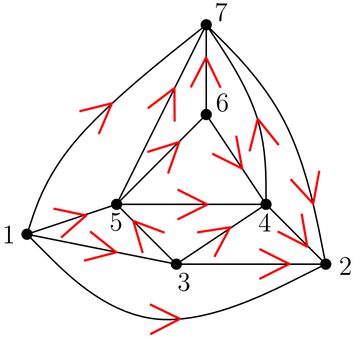

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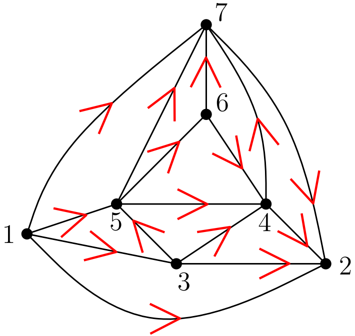
3. Topological order $x : V \to \{1 \cdots n\}$ s.t. if $u \to v$ then $x(u) < x(v)$

$x(1) < x(3) < x(5) < x(6) < x(4) < x(7) < x(2)$



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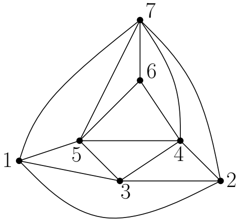

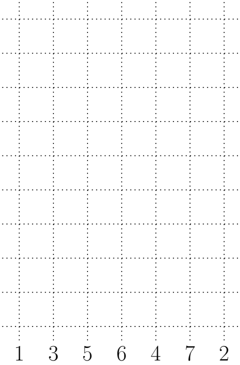

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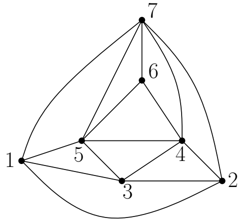

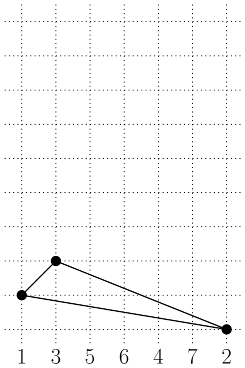

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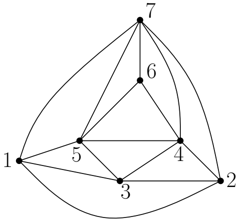

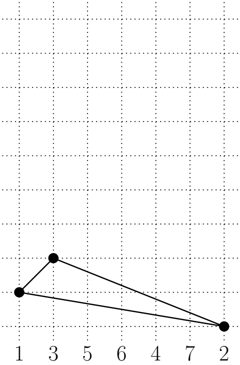

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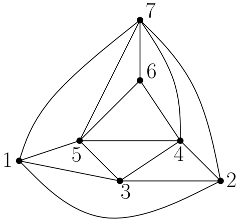

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$y(v_{k+1})$ is the smallest possible such that:

- v_{k+1} can see all its precedessors
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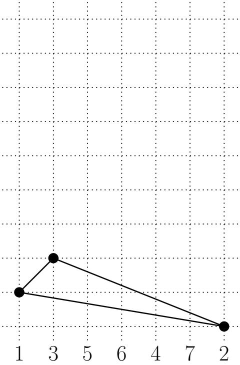

Non-aligned drawing on an $n \times O(n^3)$-grid

$x(1) < x(3) < x(5) < x(6) < x(4) < x(7) < x(2)$

Place v_1 at $(1, 2)$, v_3 at $(x(v_3), 3)$, v_2 at $(n, 1)$
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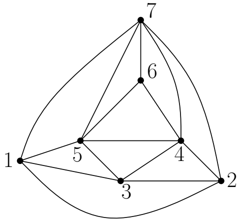
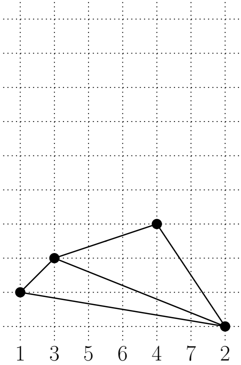
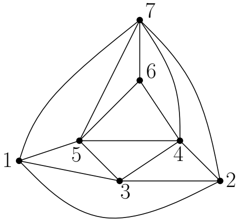




Non-aligned drawing on an $n \times O(n^3)$-grid



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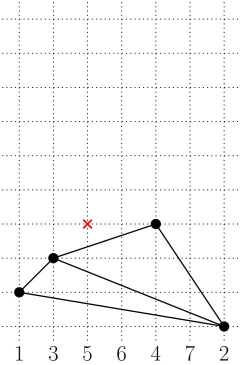

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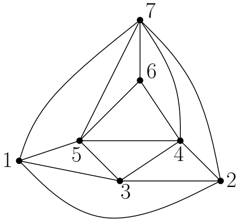

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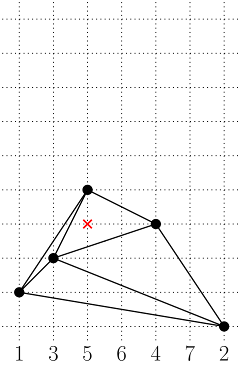

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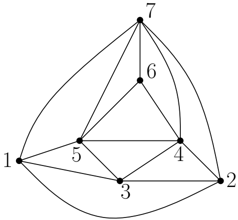

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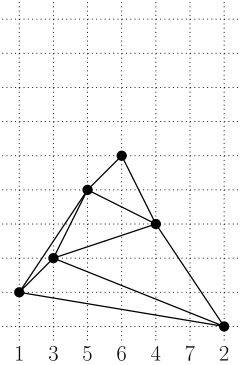

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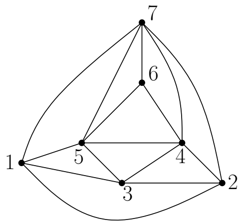

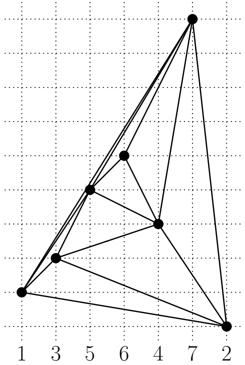

Non-aligned drawing on an $n \times O(n^3)$-grid

Left-steepness of a vertex: $s(v) = \left| \frac{y(v) - y(c_\ell)}{x(v) - x(c_\ell)} \right|$

Non-aligned drawing on an $n \times O(n^3)$-grid

Left-steepness of a vertex: $s(v) = \left| \frac{y(v) - y(c_\ell)}{x(v) - x(c_\ell)} \right|$

In the non-aligned drawing of G_k, $s(v_k) \leq \frac{(k-1)(k-2)}{2}$ for $k \geq 3$.

Non-aligned drawing on an $n \times O(n^3)$-grid

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$s(v_n) \leq \frac{1}{2}(n-1)(n-2)$

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$c_\ell = v_1$

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$c_\ell = v_1 \rightarrow x(v_n) - x(v_1) \leq n - 2$ and $y(v_1) = 2$

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$y(v_n) \leq 2 + \frac{1}{2}(n-1)(n-2)^2$

Non-aligned drawing on an $n \times O(n^3)$-grid

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Every planar graph with n vertices has a non-aligned straight-line drawing in an $n \times \left(2 + \frac{1}{2}(n-1)(n-2)^2\right)$ grid.

Our results

Every planar graph with n vertices has a:

- non-aligned drawing in a $n \times n$-grid with $\leq \frac{2n-5}{3}$ bends.
  (only 1 if the graph is 4-connected)
- non-aligned straight-line drawing in an $n \times O(n^3)$ grid
- **non-aligned straight-line drawing in an $O(n^2) \times O(n^2)$ grid**

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

Mapping $v \in V(G)$ to a point $\left(p_1(v), p_2(v), p_3(v)\right)$

Lexicographic order: For vertices u, v and $i = 0, 1, 2$, $p_i(u) <_{lex} p_i(v)$ if either $p_i(u) < p_i(v)$ or $p_i(u) = p_i(v)$ and $p_{i+1}(u) < p_{i+1}(v)$.

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

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Weak barycentric representation of G:

- $p_0(v) + p_1(v) + p_2(v) = c$ for every vertex v
- for each edge (u, v) and each vertex $w \neq \{u, v\}$, there is k s.t. $p_k(u), p_k(v) <_{lex} p_k(w)$.

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

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[Schnyder 90] Every maximal planar graph G has a straight-line planar drawing on a grid with $n-1$ rows and columns where coordinates are given by a weak barycentric representation of G.

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

$p'_i(v) := (n-1) \times p_i(v) + p_{i+1}(v)$, for $i = 0, 1, 2$, is also a weak barycentric representation.

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

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  → **planar straight-line** drawing

NON-ALIGNED DRAWING ON AN $O(n^2) \times O(n^2)$-GRID

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  → **planar straight-line** drawing
- $1 \leq p_i(v) \leq n-2 \rightarrow p'_i(v) \leq (n-1)(n-2) + (n-2) = n(n-2)$
  → drawing on an $\big(\boldsymbol{n(n-2) \times n(n-2)}\big)$-grid

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

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- $p'_i(u) \neq p'_i(v)$ for any vertices u, v and any i $\rightarrow$ **non-aligned** drawing

Non-aligned drawing on an $O(n^2) \times O(n^2)$-grid

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Every planar graph with n vertices has a non-aligned straight-line drawing on an $(n(n-2) \times n(n-2))$ grid.

And now?

Open questions:

- Find a planar graph needing more than one bend
- There is likely a better bound on the $n \times O(n^3)$ result (equation on the slopes is not tight)
- Find a planar graph needing n columns and more than $n+1$ rows

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Thank you!